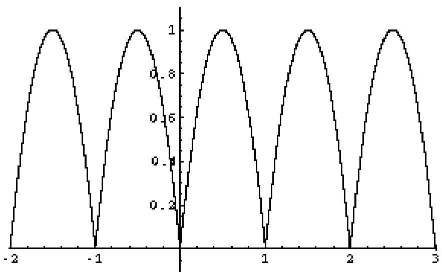
4.4 Graphs of Sine and Cosine Functions

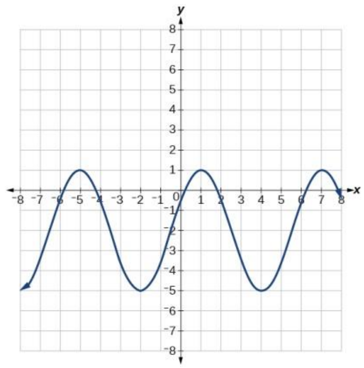
If the values of a function are the same for each given interval of the domain, the function is said to be **periodic**. The interval is the **period** of the function. (smallest interval of x that contains one copy of the repeating pattern)

Ex 1: Determine if each function is periodic. If so, state the period.

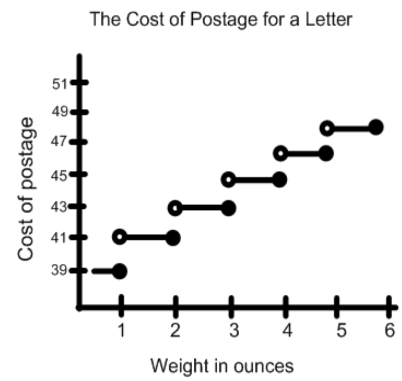
a)



b)

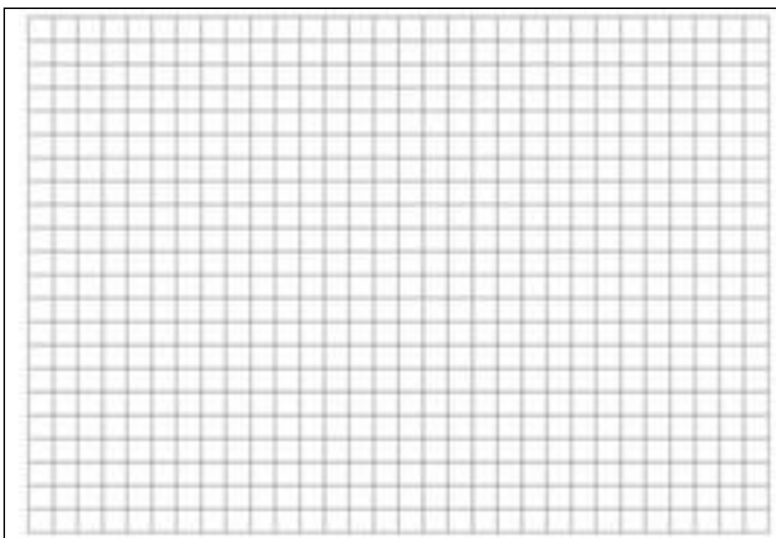


c)



Ex 2: Graph the functions $y = \sin\theta$ and $y = \cos\theta$ from -2π to 2π in multiples of $\frac{\pi}{4}$.

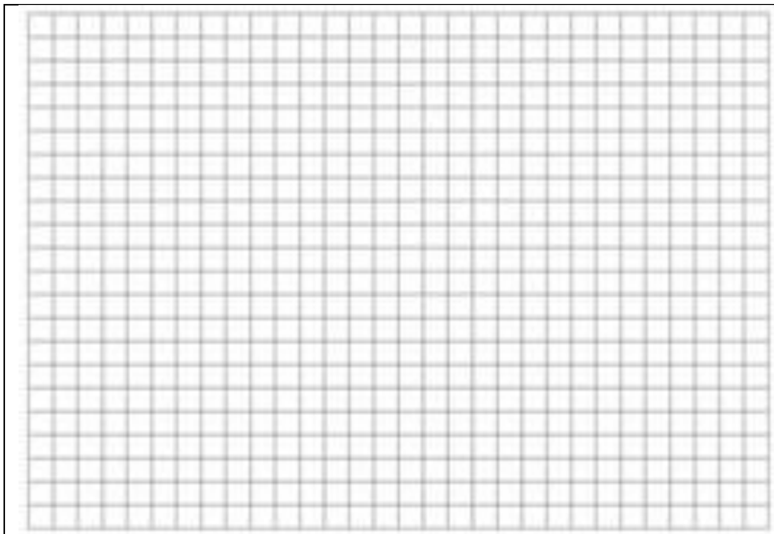
θ	-2π	$\frac{-7\pi}{4}$	$\frac{-3\pi}{2}$	$\frac{-5\pi}{4}$	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
sin θ																	



Properties of the Graph y = sin θ

1. The period is ________.
2. The domain is the set of real numbers.
3. The range is the set of real numbers between -1 and 1, inclusive.
4. The x-intercepts are located at π n, where n is an integer.
5. The y-intercept is 0.
6. The maximum values are y = 1 and occur when x = $\frac{\pi}{2} + 2\pi$n, where n is an integer.
7. The minimum values are y = -1 and occur when x = $\frac{3\pi}{2} + 2\pi$n, where n is an integer.

θ	-2π	$\frac{-7\pi}{4}$	$\frac{-3\pi}{2}$	$\frac{-5\pi}{4}$	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cos θ																	



Properties of the Graph y = cos θ

1. The period is ________.
2. The domain is the set of real numbers.
3. The range is the set of real numbers between -1 and 1, inclusive.
4. The x-intercepts are located at $\frac{\pi}{2} + \pi$n, where n is an integer.
5. The y-intercept is 1.
6. The maximum values are y = 1 and occur when x = π n, where n is an even integer.
7. The minimum values are y = -1 and occur when x = π n, where n is an odd integer.

You will be studying the graphic effect of each of the constants *a, b, c,* and *d* in the equations of the forms:

$$y = d + a\sin(bx - c) \qquad \textbf{and} \qquad y = d + a\cos(bx - c).$$

The constant factor *a* in $y = a\sin x$ acts as a scaling factor – a _______________ _______________ or _______________ _______________ of the basic sine curve.

When $|a| > 1$, the basic sine curve is _______________.

When $|a| < 1$, the basic sine curve is _______________.

The result of this is that the graph of $y = a\sin x$ ranges between _____ and _____ instead of between 1 and -1.

The absolute value of *a* is called the <u>amplitude</u> of the function $y = a\sin x$.

The range of the function $y = a\sin x$ for $a > 0$ is $-a \le y \le a$.

Definition of Amplitude of Sine and Cosine Curves

The **amplitude** of $y = a\sin x$ and $y = a\cos x$ represents half the distance between the maximum and minimum values of the function and is given by:

Amplitude= $|a|$. *amplitude can never be negative

When sketching the graphs of the basic sine and cosine functions by hand use the **five key points** in one period of each graph (intercepts, maximum, and minimum).

<u>Ex:1</u> a.) Sketch the graph of $y = 3\cos x$ on the interval $\left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$.

b.) Sketch the graph of $y = 2\sin x$ **on the interval** $[-\pi, 4\pi]$**.**

<u>**Ex:2**</u> **On the same coordinate axes, sketch the graphs of** $f(x) = 2\sin x$ and $g(x) = \frac{1}{3}\sin x$ **for one full cycle of output values** $[0, 2\pi]$**.**

We know that the graph of $y = -f(x)$ **is a** <u>**reflection**</u> **in the x-axis of the graph of** $y = f(x)$**. Therefore, the graph of** $y = -3\cos x$ **is a reflection in the x-axis of the graph of** $y = 3\cos x$**.**

Because $y = a\sin x$ **completes one cycle from** $x = 0$ **to** $x = 2\pi$**, it follows that** $y = a\sin bx$ **completes one cycle from** $x = 0$ **to** $x = \frac{2\pi}{b}$**, where** ***b*** **is a positive real number.**

Period of Sine and Cosine Functions

Let *b* be a positive real number. The **period** of $y = a\sin bx$ **and** $y = a\cos bx$ is given by:

Period = $\frac{2\pi}{b}$.

<u>ALWAYS LABEL BOTH AXES! CRITICAL POINTS MUST BE IDENTIFIED</u>

<u>Ex:3</u> **a.) Sketch the graph of** $y = \cos\frac{x}{2}$ **for one full cycle of output values (one period).**

b.) Sketch the graph of $y = \sin\frac{x}{3}$ **for one full cycle of output values (one period).**

Remember that with the sine and cosine functions the period = $\frac{2\pi}{b}$ where b is the coefficient of the argument.

When $0 < b < 1$, the period of $y = a\sin bx$ is ________________ ___________ 2π and represents a ________________ ________________ of the graph of $y = a\sin x$.

When $b > 1$, the period of $y = a\sin bx$ is _____________ _____________ 2π and represents a ________________ ________________ of the graph of $y = a\sin x$.

The constant *c* in the general equations $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$ creates

_____________________ _____________________ (shifts) of the basic sine and cosine curves.

The period of $y = a\sin(bx - c)$ is $\frac{2\pi}{b}$, and the graph of $y = a\sin bx$ is shifted by an amount $\frac{c}{b}$. The number $\frac{c}{b}$ is called the ________________ ________________.

Graphs of Sine and Cosine Functions

The graphs of $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$ have the following characteristics.

(Assume b > 0.)

Amplitude = $|a|$ Period = $\frac{2\pi}{b}$

The left and right endpoints of one-cycle interval can be determined by solving the equations $bx - c = 0$ and $bx - c = 2\pi$.

<u>Horizontal Translations</u>

<u>Ex:4</u> a.) Sketch the graph of $y = 2\sin\left(x - \frac{\pi}{2}\right)$ for one full cycle of output values (one period).

Label the key points on the graph.

Amp = **Per =** **Phase Shift =**

b.) Sketch the graph of $y = 2\cos\left(x - \frac{\pi}{2}\right)$ **for two full cycles of output values (two periods).**

Label the key points on the graph.

Amp = **Per =** **Phase Shift =**

<u>**Ex:5**</u> **a.) State the amplitude, period and the phase shift of** $y = 2\sin(3\pi x - 2\pi)$

Amp = **Per =** **Phase Shift =**

b.) State the amplitude, period, and the phase shift of $y = -3\cos(2\pi x + 4\pi)$

Amp = **Per =** **Phase Shift =**

The final type of transformation is the ____________________ ____________________ caused by the constant *d* in the equations

$$y = d + a\sin(bx - c) \qquad \textbf{and} \qquad y = d + a\cos(bx - c).$$

The shift *d* units up for $d > 0$ and *d* units down for $d < 0$. In other words, the graph oscillates about the horizontal line $y = d$ instead of about the *x*-axis.

<u>Vertical Translations</u>

<u>Ex:6</u> **Sketch the graph of $y = 4 - \cos 3x$ for two full cycles of output values (two periods).**

Label the key points on the graph.

Amp = **Per =** **Phase Shift =** **Vertical Shift =**